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On Homogeneous Locally Conical Spaces

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Theorem: Every homogeneous locally conical connected separable metric space other than S^1 or $\mathbb{R}$ is strongly n-homogeneous for each $n \geq 2$ and countable dense homogeneous. (Furthermore, countable dense homogeneity can be proven without assuming the space is connected.)

Corollary 1: If X is a homogeneous compact suspension, then X is an absolute suspension (i.e., for any two distinct points p and q of X, there is a homeomorphism from X to a suspension that maps p and q to the suspension points).

Corollary 2: If there exists a locally conical counterexample X to the Bing-Borsuk Conjecture (i.e., X is a locally conical homogeneous Euclidean neighborhood retract that is not a manifold), then X is strongly n-homogeneous for all $n \geq 2$ and countable dense homogeneous.

Relevant Definitions: Let X be a topological space. X is locally conical if every point of X has an open neighborhood that is homeomorphic to an open cone over a compact space. X is homogeneous if for any two points p and q of X, there is a homeomorphism of X that maps p to q. More generally, for $n \geq 1$, a space X is strongly n-homogeneous if every bijection between two n-element subsets of X can be extended to a homeomorphism of X. X is countable dense homogeneous if for any two countable dense subsets A and B of X, there is a homeomorphism of X which maps A onto B.

Discontinuous Homomorphisms and other inscrutable objects

Michael Andersen
Brigham Young University

A standard tool in algebraic topology is to pass between a continuous map between spaces and the corresponding homomorphism of fundamental groups using the π_1 functor. It is a non-trivial question to ask when a specific homomorphism is induced by a continuous map; that is, what is the image of the π_1 functor on homomorphisms?

Conner and Spencer proved that there exist homomorphisms that are not induced by continuous functions between topological spaces. We use methods from Shelah and Pawlikowski to prove that Conner and Spencer could not have constructed these homomorphisms with a weak version of the Axiom of Choice. This leads us to define and examine a class of pathological objects that cannot be constructed without a strong version of the Axiom of Choice, which we call the class of inscrutable objects.

On a rational analogue of a conjecture of Singer

Grigori Avramidi
The Ohio State University

For a closed manifold M with contractible universal cover, the Singer conjecture predicts that the L^2-Betti numbers of M are concentrated in the middle dimension. In this talk, I will discuss a construction of closed manifolds with rationally acyclic universal covers whose L^2-Betti numbers are not concentrated in the middle dimension.

On the scarcity of crumpled cube sewings that yield the n-sphere.

Robert Daverman
University of Tennessee Knoxville

For $n \geq 5$ we produce a crumpled n-cube C and homeomorphism h of BdC to itself such that, for any homeomorphism $H : \text{Bd}C \to \text{Bd}C$ sufficiently close to h, the sewing space $C \cup_H C$ is a non-manifold. This contrasts starkly with a classical 3-dimensional result that a dense collection of sewings of an arbitrary pair of crumpled 3-cubes yields the 3-sphere. The key new ingredient is recent work by S. Krushkal, providing a Cantor set in the n-sphere, $n \geq 4$, that cannot be slipped off itself with a small ambient homeomorphism.

Hidden symmetries of knot complements

Jason DeBlois
University of Pittsburgh

Do there exist hyperbolic knot complements in the three-sphere that have hidden symmetries, other than the three known examples? Sadly, I will not answer this question. But I will try to motivate it, in particular defining hidden symmetries, and describe some related results.

Topological Linear Algebra: More Matrix Madness

Greg Friedman*, Efton Park
**Texas Christian University*

We will discuss some new results in ongoing work with Efton Park (TCU) to classify the unitary equivalence classes of normal matrices with coefficients in the ring $C(X)$ of continuous complex-valued functions on the space X. Such matrices can be thought of as bundles of complex-valued matrices over X, and their properties depend in interesting ways on the algebraic-topological properties of X.

On the topology of spaces admitting a coaxial $\mathbb{Z}$-action

Ross Geoghegan, Craig Guilbault*, Michael Mihalik
University of Wisconsin-Milwaukee

In 2012 Geoghegan and Guilbault improved upon a theorem by David Wright asserting that, when a one-ended, simply connected, locally finite polyhedron X with *pro-monomorphic* fundamental group at infinity admits a proper $\mathbb{Z}$-action, then the fundamental group at infinity of X is (up to pro-isomorphism) an inverse sequence of finitely generated free groups. Using a rather indirect argument, Guilbault and Geoghegan were able to add π_1-semistability to the conclusions, thereby showing that X must have a *stable* finitely generated free fundamental group at infinity.

In this talk, we will describe further improvements to this theorem. Most notably, the pro-monomorphic hypothesis is weakened to the existence of a *coaxial* homeomorphism $j : X \to X$ generating the $\mathbb{Z}$-action. Under that hypothesis we still obtain all of the above conclusions, aside from the pro-monomorphic part (which was true only by hypothesis). The new proof is more direct and more geometric; moreover, it provides additional detail and a clearer picture of the fundamental group at infinity. At the same time, it applies to a much broader class of examples and has new implications for geometric group theory.

Investigating Large Scale Properties by Mapping to Small Scale Spaces

Michael Holloway
University of Tennessee

Large scale geometry studies the properties of a space which persist as it is viewed from farther and farther away. In this talk, we discuss two types of maps to small scale spaces which can be used to investigate large scale properties of a space.

Quasimorphisms on groups that act on trees

Joel Louwsma
Niagara University

We construct efficient quasimorphisms on groups that act on trees and show that their defect is at most 6. Calculations in the Baumslag-Solitar group $BS(2,3)$ show that this is the smallest possible defect that can be achieved in this generality. A consequence of our result is that every suitable element of a group that acts on a tree must have stable commutator length at least $1/12$. In Baumslag-Solitar groups, we show that no element can have stable commutator length between 0 and $1/12$. This is joint work with Matt Clay and Max Forester.

Counting embeddings

Fedor Manin
University of Toronto

How many isotopy classes of reasonable embeddings are there between manifolds M and N? How does this number grow as we let the embeddings get crazier? The answers depend a lot on the measure of craziness as well as on the category and codimension of the manifolds, but a particularly interesting case is that of PL manifolds in codimension at least 3. In this setting, there is local unknotting, and so any nontrivial behavior is necessarily non-local. If M and N are simply connected, this means that we can get polynomial bounds on the number of embeddings in terms of a bound on the bilipschitz constant. Showing this requires a generalization of equivariant rational homotopy theory to a "rational homotopy theory for diagrams."

Two Metrics of J. Nagata in Coarse Geometry

Atish Mitra
Montana Tech (University of Montana)

In the '60s, J. Nagata introduced two metrics which were used to characterize covering dimension. In the past few years, those metrics have been used in the study of coarse geometry. In this talk we discuss a few of those recent results.

The Strong Atiyah Conjecture and computations of L^2-Betti numbers

Wiktor Mogilski
Binghamton University

The Strong Atiyah Conjecture predicts that for any group G with bounded torsion, the L^2-Betti numbers of any G-space are rational, with denominators determined by the order of the torsion subgroups. In this talk we will restrict ourselves to the setting of Coxeter groups, and I will present a special trick that, in many cases, improves the Strong Atiyah Conjecture prediction of the denominators of the L^2-Betti numbers. In many examples, this improvement (along with additional work) allows us to make complete computations of the L^2-Betti numbers. I will conclude by exploiting this trick to obtain new affirmative results regarding the Singer Conjecture for Coxeter groups. This is joint work with Kevin Schreve.

More Examples of Pseudo-Collar Structures on High-Dimensional manifolds

Jeffrey Roland
Marquette University

In "A Geometric Reverse to The Plus Construction and Some Examples of Pseudo-Collars on High-Dimensional Manifolds", uncountably many distinct ends of manifolds called pseudo-collars, which are "stackings" of 1-sided h-cobordisms, were produced. Each pseudo-collars had the same boundary and pro-homology systems at infinity and similar group-theoretic properties for their pro-fundamental group systems at infinity. In particular, the kernel group of each group extension for each 1-sided h-cobordism in the pseudo-collars was the same group, a free product of 2 copies of Thompson's group V.

In this talk, we extend this construction to have the kernel groups of each group extension be a free product $K * K$ of any finitely presented, Hopfian, centerless, superperfect group K which contains a countably infinte list of elements $\{a_1, a_2, a_3, \ldots\}$ with the property that for any isomorphism $\phi : K \to K$, $\phi(a_i) \neq a_j^{\pm 1}$ for $i \neq j$. Note that this class of groups includes the fundamental group of any closed hyperbolic manifold M with first two homology groups $H_1(M) = H_2(M) = 0$. Note futher that the $(1, n)$ Dehn filling of the figure-8 knot complement in $\mathbb{S}^3$ for $n > 1$ satisfies this condition, so already we have a countably infinite collection of such kernel groups, for each of which we produce an uncountable collection of distinct pseudo-collars.

The notion of pseudo-collars originated in Hilbert cube manifold theory, where it was part of a necessary and sufficient condition for placing a $\mathcal{Z}$-set as the boundary of an open Hilbert cube manifold. We are interested in pseudo-collars on finite-dimensional manifolds for the same reason, attempting to put a $\mathcal{Z}$-set as the boundary of an open high-dimensional manifold.

New Methods for G-acyclic Resolutions in Cohomological Dimension

Leonard R. Rubin
University of Oklahoma
Vera Tonić
University of Rijeka

The Edwards-Walsh cell-like resolution theorem states that for all $n \in \mathbb{N}$ and every compact metrizable space X with $\dim_{\mathbb{Z}} X \leq n$, there exists a compact metrizable space Z with $\dim Z \leq n$ and a cell-like map of Z onto X. This generated a lot of interest in resolutions of a similar nature, and eventually A. Dranishnikov proved the $\mathbb{Z}/p$-resolution theorem and M. Levin proved the $\mathbb{Q}$-resolution theorem. These went as follows. If $G \in \{\mathbb{Z}/p, \mathbb{Q}\}$ and a compact metrizable space X has $\dim_G X \leq n$, then there exists a compact metrizable space Z with $\dim Z \leq n$ and a G-acyclic map of Z onto X ($n \geq 2$ in case $G = \mathbb{Q}$).

In all three of the proofs, the space X was represented as the limit of an inverse sequence of finite triangulated polyhedra, and a significant part of the proofs required the construction of complicated, abstruse extensions built upon the n-skeleta of these polyhedra. Our aim is to present proofs of these (and other) resolution theorems by a new method that in the case of the cell-like resolution theorem requires no extensions at all, in the $\mathbb{Z}/p$-resolution theorem uses only Moore spaces, and in the $\mathbb{Q}$-resolution theorem uses only the $(n+1)$-skeleton of the Eilenberg-MacLane complex $K(\mathbb{Q}, n)$. We obtain a resolution theorem stronger than the one for $\mathbb{Q}$ via the latter approach.

ℓ^2-Betti numbers and graphs

Timothy Schroeder
Murray State University

This talk will explain the connection between ℓ^2-homology and graphs, and will describe a (complicated?!?) program for using ℓ^2-technology to estimate the genus of a graph.

Spanning Trees and Mahler Measure

Daniel Silver
University of South Alabama

Infinite periodic graphs, graphs that are invariant under translation in one or more independent directions, are of interest in crystallography and statistical mechanics. One measure of complexity for such a graph is the spanning tree entropy, the exponential growth rate of the number of spanning trees in a sequence of finite subgraphs approximating the whole graph. This entropy has been calculated using mainly combinatoric and analytic arguments.

Using ideas of algebraic dynamics, we give a simplified approach to showing that the spanning tree entropy is the Mahler measure of a Laplacian polynomial that is easily obtained from graph data. We discuss applications to knot determinants. We reformulate Lehmer's question about polynomials in these terms.

Two shall be the number of the counting

Eric Swenson
Brigham Young University

Let Z be a CAT(0) boundary of a one-ended group. If Z can be separated by removing a finite set, then Z has a cut pair (and no cut points).

Cocompact Cubulations of Mixed Manifolds

Joseph Tidmore
University of Wisconsin-Milwaukee

Recent breakthroughs in the study of hyperbolic 3-manifolds came by showing their fundamental groups have a property called virtually compact special. This means their fundamental groups act properly, cocompactly, and in a "special" way on a CAT(0) cube complex. A natural outgrowth is to classify which 3-manifold groups are virtually compact special. This question is solved in every case except for what is called a mixed manifold.

From geometrization we know that if a prime 3-manifold does not admit a geometric structure, it can be cut along embedded incompressible tori called JSJ tori so that each component of the cut-open manifold is either hyperbolic or Seifert fibered. Such a 3-manifold is called a mixed manifold if it has at least on JSJ torus and its decomposition has at least one hyperbolic component. We show that a mixed manifold group is virtually compact special iff the Seifert fiber components of the JSJ decomposition satisfy an algebraic condition related to how it is attached to its neighbors.

The topology of the Jones polynomial

Anh Tran
University of Texas at Dallas

We will discuss old and new conjectures about the topology of the Jones polynomial. These include the AJ conjecture, the slope conjecture, and the strong slope conjecture. The AJ conjecture of Garoufalidis relates the A-polynomial and the colored Jones polynomial of a knot. The A-polynomial was introduced by Cooper et al. in 1994 and has been fundamental in geometric topology. A similar conjecture to the AJ conjecture was also proposed by Gukov from the viewpoint of the Chern-Simons theory. The slope conjecture of Garoufalidis and two new conjectures of Kalfagianni and myself are about the relationship between the degree of the colored Jones polynomial of a knot and the topology of the knot. These conjectures assert that certain boundary slopes and Euler characteristics of essential surfaces in a knot complement can be read off from the degree of the colored Jones polynomial.

On spaces with connected Higson coronas

Thomas Weighill
University of Tennessee

Coarse geometry is the study of large-scale properties of metric spaces. The main motivation for the study of such properties comes from geometric group theory and higher index theory. One way to investigate the large-scale properties of a metric space is to study its Higson corona, a compact topological space which captures large-scale behavior of the space. In this talk we exhibit a connection between a topological property of the Higson corona (namely, connectedness) and a natural categorical condition stated in the coarse category. This result motivates the study of categorical conditions in the coarse category. As a further result along these lines, we give a coarse categorical characterization of the ω-excisive decompositions introduced by Higson, Roe and Yu. We also characterize connectedness of the Higson corona in terms of coarse cohomology, and look at some special cases and generalizations of the main result.

Periodic plane graphs and medial link components

Susan Williams
University of South Alabama

Let G be a connected, locally finite plane graph with a free $\mathbb{Z}^2$-action by automorphisms. Applying the medial link construction to G, we obtain a (generalized) link of infinitely many components, which may be unbounded. We associate to G a finitely generated module $C(G)$ over the polynomial ring $F2[x^{\pm 1}, y^{\pm 1}]$, the mod 2 coloring module. The orbit structure of the components under $\mathbb{Z}^2$ can be determined from the sequence of elementary divisors of $C(G)$.