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The topology of the moduli space of curves

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The moduli space of curves plays a fundamental role in many areas of math, ranging from low-dimensional topology and geometry to mathematical physics. Because it lies at the juncture of so many areas of mathematics, it can be described from many points of view. In my lectures I will give an introduction to different ways to think about it, and a broad survey of what we know and don't know about its topology.

Rigidity of locally symmetric rank one manifolds of infinite volume

Boris Apanasov
University of Oklahoma

We create some analogue of the Sierpiński carpet for nilpotent geometry on horospheres in symmetric rank one negatively curved spaces $H^n_{\mathbb{F}}$ over division algebras $\mathbb{F} \neq \mathbb{R}$, i.e over complex $\mathbb{C}$, quaternionic $\mathbb{H}$, or octonionic/Cayley numbers $\mathbb{O}$. The original Sierpiński carpet in the plane was described by Wacław Sierpiński in 1916 as a fractal generalizing the Cantor set.

Making such a Sierpiński carpet with a positive Lebesgue measure at the sphere at infinity $\partial H^n_{\mathbb{F}}$ and defining its "stretching", we construct a non-rigid discrete $\mathbb{F}$-hyperbolic groups $G \subset \operatorname{Isom} H^n_{\mathbb{F}}$ whose limit set $\Lambda(G)$ is the whole sphere at infinity $\partial H^n_{\mathbb{F}}$. This answers questions by G.D.Mostow [Mostow, 1973], L.Bers [Bers, 1974] and S.L.Krushkal [Krushkal, 1984] about uniqueness of a conformal or CR structure on the sphere at infinity $\partial H^n_{\mathbb{F}}$ compatible with the action of a discrete group $G \subset \operatorname{Isom} H^n_{\mathbb{F}}$.

Previously, for the real hyperbolic spaces, this problem was solved by Apanasov [Apanasov, 1978, 2000]. Due to D.Sullivan [Sullivan, 1981] rigidity theorem generalized by Apanasov [Apanasov, 2000] and [Apanasov, to appear], Theorem 5.19, the complement of the constructed class of discrete groups $G \subset \operatorname{Isom} H^n_{\mathbb{F}}$ (having a positive Lebesgue measure of the set of vertices of its fundamental polyhedra at infinity) whose limit set $\Lambda(G)$ is the whole sphere at infinity $\partial H^n_{\mathbb{F}}$ consists of groups rigid in the sense of Mostow.

Coarse Homotopy Extension Property

William Braubach
University of Wisconsin-Milwaukee

I will discuss the notions of a coarse homotopy between coarse maps and the coarse homotopy groups. I will also describe coarse versions of CW complexes and the mapping cylinder construction and show that they have the coarse homotopy extension property. These results lead to a coarse version of Whitehead's theorem: if a coarse map between coarse CW complexes induces an isomorphism on the coarse homotopy groups, then the map is a coarse homotopy equivalence.

C^2 Morse functions on $\overline{M_{g,n}}$

Changjie Chen
Brown University

I will introduce a family of C^2 Morse functions on the Deligne-Mumford compactification of the moduli space. This is the first example of such functions.

On Cohomological Dimension of Group Homomorphisms

Aditya De Saha
University of Florida

The (co)homological dimension of a group homomorphism is defined as the highest number n such that the induced map on (co)homologies is nontrivial at the n'th dimension, with some coefficients. In the talk, I will present a couple of results relating cohomological dimension with homological dimension, and the cohomological dimension of products of homomorphisms. This is joint work with Dr. Alexander Dranishnikov.

The Burau representation and shapes of polyhedra

Ethan Dlugie
University of California Berkeley

The Burau representation of braid groups has been around for almost a century. Still we don't know the full answer to whether this representation is faithful. The only remaining case is for the $n = 4$ braid group, and faithfulness here has intimate connections to the question of whether the Jones polynomial detects the unknot. By in essence specializing the t parameter in this representation to certain roots of unity, an interesting connection appears with the moduli space of flat cone metrics on spheres explored by Thurston. Leveraging this connection, I will explain how one can place strong restrictions on the kernel of the $n = 4$ Burau representation.

Periodic points of translation surfaces

Sam Freedman
Brown University

Translation surfaces are surfaces that are flat except potentially at a finite set of points with conical singularities, such as a square torus. An important class of translation surfaces are Veech surfaces: those whose (affine) automorphism group is as large as possible. While a generic point of a Veech surface equidistributes under the action of its automorphism group, there is an exceptional finite set of points with finite orbits. These periodic points appear throughout Teichmller dynamics, such as in counting holomorphic sections of families of Riemann surfaces and in blocking problems on billiard tables. In this talk we first describe joint work with Zawad Chowdhury, Samuel Everett and Destine Lee that gives an algorithm that computes the set of periodic points for a given Veech surface. We then describe work that classifies the periodic points of Prym eigenforms, an infinite family of Veech surfaces in genera 2, 3 and 4.

A generalized theory of expansions and collapses

Craig Guilbault
University of Wisconsin-Milwaukee

A theorem from Dan Gulbrandsen's dissertation, about Z-compactifications of CAT(0) cube complexes, led us to consider general questions on the use of inverse limits to obtain nice compactifications. One first considers generic inverse sequences of compacta in which the bonding maps are retractions, and is then led to a special type of retraction which we call a *topological collapse*.

I will review some classical examples of inverse limit spaces and some related issues regarding topologies on a simplicial complexes. I will also recall the classical theory of expansions and collapses and extend those notions to topological versions. Finally, I will tie these topics to the problem of obtaining *Z-compactifications* of certain spaces, such as CAT(0) cube complexes. Our work has connections to work by Chapman, Siebenmann, Ferry, Marsh, Prajs and others, which will be touched upon. Joint with Dan Gulbrandsen.

The Anatomy of Ghastly Spaces and the Disjoint Homotopies Property

Denise Halverson
Brigham Young University

Ghastly spaces are resolvable generalized manifolds that contain no embedded j-cells for $1 < j < n$. With sufficient care in the construction of a ghastly space, the resulting space can be shown to be a codimension one manifold factor. The disjoint homotopies property is a sufficient general position property to detect codimension one manifold factors. The specific anatomy of ghastly spaces that gives rise to the disjoint homotopies property will be illuminated in this presentation.

From Separating Sets to Cube Complexes

Matthew Haulmark
Binghamton University

Let M be a Peano continuum and suppose that G acts on M by homeomorphisms. Using the cut points of M one can obtain a tree on which G acts (Bowditch, Swenson). Similarly, certain cut pairs of M can also be used to get a tree on which G acts (Bowditch, Papasoglu-Swenson). In this talk we will discuss getting an action on a cube complex from the action of G on a connected locally path-connected metric space with separating sets satisfying certain conditions. We will also explore a few consequences of this result. This project is joint with Jason Manning.

Right-angled Coxeter Groups with Menger Curve Boundary

Cong He
University of Wisconsin-Milwaukee

Hyperbolic Coxeter groups with Sierpinski carpet boundary was investigated by Swiatkowski. And hyperbolic right-angled Coxeter group with Gromov boundary as Menger curve was studied by Daniel Danielski. Also, Haulmark, Hruska, and Sathayes produced the first known examples of nonhyperbolic CAT(0) groups whose visual boundary is homeomorphic to the Menger curve. The examples in question are the Coxeter groups whose nerve is a complete graph on n vertices for n greater than or equal to 5. Recently, Danielski and Swiatkowski gave complete characterizations (in terms of nerves) of the word hyperbolic Coxeter groups whose Gromov boundary is homeomorphic to the Sierpinski curve and to the Menger curve, respectively. In our presentation, we find new examples with both hyperbolic 4 and nonhyperbolic groups which state: Genus one surface with one boundary component admits a flag triangulation L such that W_L has a Menger curve boundary, and W_L can be non-hyperboic. The construction in Haulmark, Hruska, and Sathayes paper depended on a slight extension of Sierpinskis theorem on embedding 1dimensional planar compacta into the Sierpinski carpet. However, our methods depend on a perturbing trick for paths and special techniques for nullity condition; also, we exploit good properties of Pontryagin surface.

Symplectic geometry of Anosov flows in dimension 3

Surena Hozoori
University of Rochester

Since their introduction in the early 1960s, Anosov flows have defined an important class of dynamics, thanks to their many interesting chaotic features and rigidity properties. Moreover, their topological aspects have been deeply explored, in particular in low dimensions, thanks to the use of foliation theory in their study. Although the connection of Anosov flows to contact/symplectic geometry was noted in the mid 1990s by Mitsumatsu and Eliashberg-Thurston, such interplay has been left mostly unexplored. I will discuss recent developments in the interactions of the two theories based on purely symplectic geometric characterization of Anosov 3-flows.

n-knots in $S^n \times S^2$ and contractible $(n+3)$-manifolds

Geunyoung Kim
University of Georgia

In 1961, Mazur constructed a contractible, compact, smooth 4-manifold with boundary which is not homeomorphic to the standard 4-ball, using a 0-handle, a 1-handle and a 2-handle. In this talk, for any integer $n \geq 2$, we construct a contractible, compact, smooth $(n+3)$-manifold with boundary which is not homeomorphic to the standard $(n+3)$-ball, using a 0-handle, an n-handle and an $(n+1)$-handle. The key step is the construction of an interesting knotted n-sphere in $S^n \times S^2$ generalizing the Mazur pattern.

On LS-category of group homomorphism with torsion

Nursultan Kuanyshov
University of Florida

The Lusternik-Schnirelmann category (LS-category) is a numerical homotopy invariant of a topological space. Nevertheless, the definition of LS-category can be extended to discrete groups. In the 50s Eilenberg and Ganea proved that the LS-category of a discrete group Γ, $cat(\Gamma) = cd(\Gamma)$. Jamie Scott conjectured the same for discrete group homomorphism $\phi : \Gamma \to \Lambda$, $cat(\phi) = cd(\phi)$. In this talk we discuss the recent progress on this conjecture. In particular, why the conjecture holds for homomorphisms arbitrary finite groups and for homomorphisms between finitely generated abelian groups.

Embeddings of Spaces of Persistence Diagrams

Atish Mitra
Montana Tech

Persistence diagrams are topological summaries arising from viewing point clouds through the lens of homological algebra. A key problem of practical interest is quantifying distortions of embeddings in Hilbert space - of spaces of persistence diagrams with appropriate metrics. We will discuss progress made in this direction.

A discrete four vertex theorem for hyperbolic polygons

Wiktor Mogilski
Utah Valley University

There are many four vertex type theorems appearing in the literature, coming in both smooth and discrete flavors. The most familiar of these is the classical theorem in differential geometry, which states that the curvature function of a simple smooth closed curve in the plane has at least four extreme values. This theorem admits a natural discretization to Euclidean polygons due to O. Musin. In this article we adapt the techniques of Musin and prove a discrete four vertex theorem for convex hyperbolic polygons.

Random quotients and cubulation

MurphyKate Montee
Carleton College

The study of random groups via a random presentation was started by Gromov. Perhaps the most well-known result in this field is frequently phrased as “most groups are hyperbolic.” More precisely, in the Gromov model of random groups, a group is either hyperbolic or trivial with probability 1, where the probability depends on a parameter called the density. In this talk I’ll introduce the Gromov density model, explain what is known about cubulation (and Property (T)) in this model, and talk about how some of these results can be transported to a new model of random quotients of free products. The results described here come from joint work that is still in progress with E. Einstein, S. Krishna, T. Ng, and M. Steenbock.

Quantum Invariant for surface diffeomorphisms

Tushar Pandey
Texas A&M University

We will look at the volume conjecture by Bonahon, Wong and Yang, relating the quantum invariants of a self-homeomorphism of a surface coming from representation theory of the Kauffman bracket skein algebra and the hyperbolic volume of the mapping torus. We will use the Chekhov-Fock algebra to explicitly compute the invariants for self-homeomorphisms of the four punctured sphere, and prove the BWY volume conjecture for this case under some geometric/technical assumptions.

Random quotients of hyperbolic groups and Property (T)

Prayagdeep Parija
University of Wisconsin-Milwaukee

How does a random quotient of a group look like? Gromov looked at the density model of quotients of free groups. The density parameter d measures measures the proportion of the Cayley ball picked as relators. Using this model, he proved that for $d < 1/2$, a typical quotient of a free group is non-elementary hyperbolic. Ollivier extended Gromov's result to show that for $d < 1/2$ a typical quotient of even a non-elementary hyperbolic group is non-elementary hyperbolic.

Z̀uk/Kotowski-Kotowski proved that for $d > 1/3$, a typical quotient of a free group has Property (T). We show that (in a closely related density model) for $1/3 < d < 1/2$, a typical quotient of a non-elementary hyperbolic group is non-elementary hyperbolic and has Property (T). This provides an answer to a question of Gromov (and Ollivier).

Congruence subgroups of braid groups

Peter Patzt
University of Oklahoma

The congruence subgroups of braid groups arise from a congruence condition on the integral Burau representation $B_n \to GL_n(\mathbb{Z})$. We find the image of such congruence subgroups in $GL_n(\mathbb{Z})$ - an open problem by Dan Margalit. Additionally, we characterize the quotients of braid groups by their congruence subgroups in terms of symplectic congruence subgroups.

A fibering theorem for 3-manifolds

Jordan Sahattchieve
(Formerly) University of Michigan

I will present some results in the spirit of the Stallings fibration theorem.

Property R_∞ for Artin groups

Ignat Soroko
University of North Texas

A group G has property R_∞ if for every automorphism ϕ of G the number of twisted ϕ-conjugacy classes is infinite. This property is motivated by the topological fixed point theory, and has been a subject of active research. Among the groups which have this property are hyperbolic and relatively hyperbolic groups, mapping class groups, generalized Baumslag-Solitar groups and some others. However, the general picture of which groups have this property is quite elusive. In a joint project with Matthieu Calvez, we establish property R_∞ for some spherical and affine Artin groups by utilizing their close relation to certain mapping class groups of punctured surfaces.

A Note on the Question of the Splitability of Mazur Manifolds

Pete Sparks
University of Wisconsin-Milwaukee

We define a *Mazur-like manifold* analogous to the way Mazur manifolds are defined in Mazur's "A note on some contractible 4-manifolds" only varying the attaching curve. We say a compact contractible manifold with nonempty boundary M^n *splits* if $M^n = A \cup_C B$ with $A, B, C \approx \mathbb{B}^n$. We call such a pair $\{A, B\}$ a *splitting* of M^n. We are interested in the question: does the Mazur manifold Ma split as $Ma = A \cup_C B$ where $A,\ B,\ C \approx \mathbb{B}^4$? If $M^n = A \cup B$ with $A, B \approx \mathbb{B}^n$ we call the cover $\{A, B\}$ a *partial splitting.* We exhibit a necessary condition for certain covers $\{A, B\}$ of Mazur-like manifolds to be partial splittings.

Coset labelled graphs

Mathew Timm
Bradley University

We will look at a type of graph called a coset labeled graph and show how to use them to construct some topologically interesting complexes.

Coarse embeddability of Wasserstein space and the space of persistence diagrams

Thomas Weighill
Universtiy of North Carolina at Greensboro

Embeddings of non-linear data into vector spaces are important as they enable machine learning and statistical inference methods to be applied. We study embeddings of two kinds of objects: persistence diagrams arising from topological data analysis, and distributions viewed from the perspective of optimal transport. Negative results have recently been proved showing the non-existence of coarse embeddings into Hilbert space of the space of persistence diagrams and of the space of distributions with certain metrics, but there remain important unsettled cases for both. In this talk, we show that finite subsets of each of these spaces coarsely embed into the other uniformly. As a corollary, we are able to show an equivalence between the embeddability of these two spaces, uniting open questions from topological data analysis and optimal transport. This is joint work with my PhD student Neil Pritchard.